# Time-domain discontinuous Galerkin method for room acoustic modeling

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## Introduction

Room acoustics studies the sound behaviors in an enclosure. Room acoustic modeling aims to simulate the sound fields in complex enclosures by numerically calculating the impulse responses (or energy-time curves) of the space of interest.

## Objectives

This PhD project is motivated towards a better understanding and prediction of the sound propagation in open-plan spaces. In the context of scientific contributions, it aims at implementing, further developing and validating an **efficient** and **accurate** wave-based framework for modeling sound propagation in **geometry complicated** enclosure involving general acoustic boundary conditions. Specific goals:

1. To address the positioning of the discontinuous Galerkin (DG) method as a time-domain wave-based method for room acoustic modeling purposes.
2. To develop the high-order accurate modeling of **reflection** and **transmission** of acoustic waves impinging on boundaries.
3. To develop **efficient time-integration** schemes to increase the simulation efficiency for realistic problems containing geometric or parametric constraints without losing high-order accuracy.

## Accurate boundary modeling

Starting from linear acoustic equations

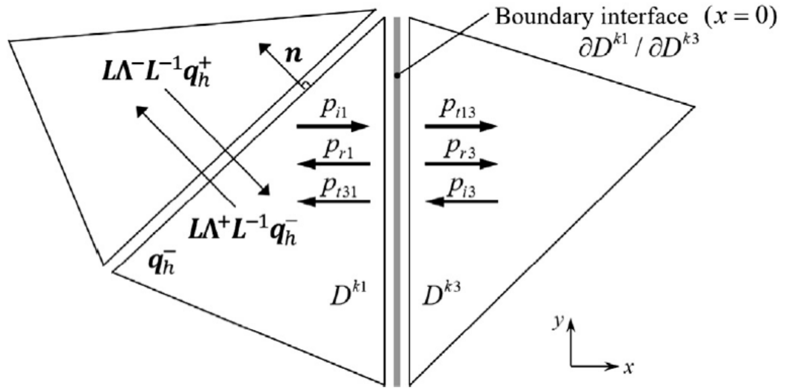
$$\frac{\partial \boldsymbol{v}}{\partial t} + \frac{1}{\rho_0}\nabla p = \boldsymbol{0}, \qquad \frac{\partial p}{\partial t} + \rho_0 c_0^2 \nabla\cdot\boldsymbol{v} = 0,$$

$$\frac{\partial \boldsymbol{q}}{\partial t} + \nabla\cdot \boldsymbol{F}(\boldsymbol{q}) = \frac{\partial \boldsymbol{q}}{\partial t} + \boldsymbol{A}_j\frac{\partial \boldsymbol{q}}{\partial x_j} = 0$$

$$\boldsymbol{q}(\boldsymbol{x},t) = [u,v,w,p]^T$$

DG discretization

$$\int_{D^k}\left(\frac{\partial \boldsymbol{q}_h^k}{\partial t} + \nabla\cdot\boldsymbol{F}_h^k(\boldsymbol{q}_h^k)\right) l_i^k \,\mathrm{d}\boldsymbol{x} = \int_{\partial D^k} \boldsymbol{n}\cdot\left(\boldsymbol{F}_h^k(\boldsymbol{q}_h^k) - \boldsymbol{F}^*(\boldsymbol{q}_h^-,\boldsymbol{q}_h^+)\right) l_i^k \,\mathrm{d}\boldsymbol{x}$$




High-order accurate and generic time-domain reflection and transmission boundary condition formulation for locally-reacting materials based on plane wave reflection coefficient $R$ and transmission coefficient $T$, and characteristic waves.

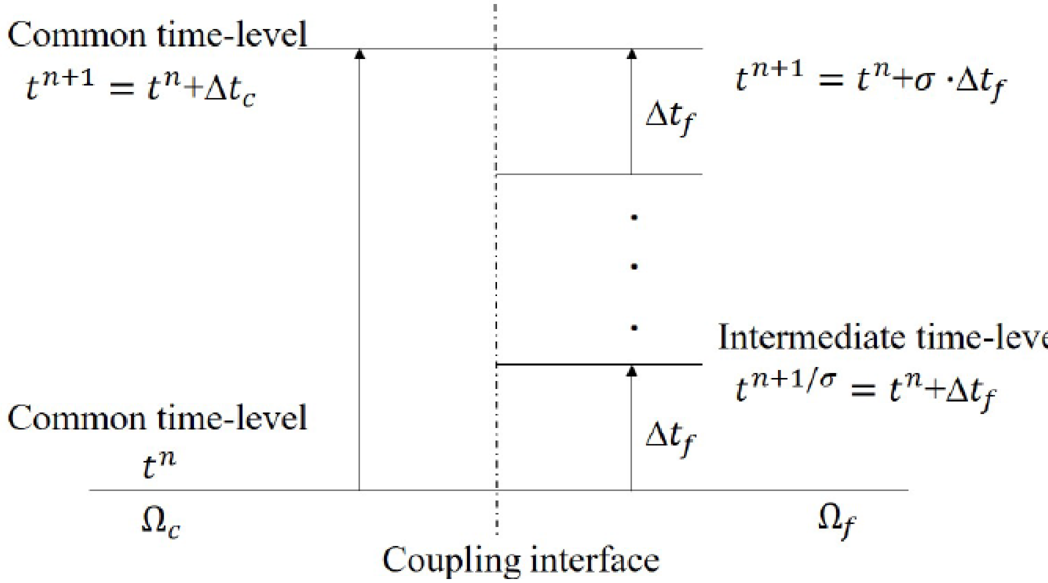
$$\boldsymbol{n}\cdot\boldsymbol{F}^*(\boldsymbol{q}_h^{1-},\boldsymbol{q}_h^{1+}) = \boldsymbol{L}(\Lambda^+\boldsymbol{L}^{-1}\boldsymbol{q}_h^{1-} + \Lambda^-\boldsymbol{L}^{-1}\boldsymbol{q}_h^{1+})$$

$$= \boldsymbol{L}\Lambda\begin{bmatrix} 0 \\ 0 \\ \varpi_{n1}^{out1-} \\ \int_{-\infty}^{t}\varpi_{n1}^{out1-}(\tau)R(t-\tau)\mathrm{d}\tau + \int_{-\infty}^{t}\varpi_{n3}^{out3-}(\tau)T(t-\tau)\mathrm{d}\tau \end{bmatrix}$$

$$\varpi_{n1}^{out1-} = p^{1-}/\rho c + \boldsymbol{v}^{1-}\cdot\boldsymbol{n}_1 \quad \text{and} \quad \varpi_{n3}^{out3-} = p^{3-}/\rho c + \boldsymbol{v}^{3-}\cdot\boldsymbol{n}_3$$

## Efficient time-integration

A novel local time-stepping approach, based on the arbitrary high-order derivatives (ADER) methodology, is proposed to increase the simulation efficiency for realistic problems containing geometric or parametric constraints is proposed.




## Results

1. H. Wang et al. *Room acoustics modelling in the time-domain with the nodal discontinuous Galerkin method*. JASA 2019.
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3. H. Wang et al. *Frequency-dependent transmission boundary condition in the acoustic time-domain nodal discontinuous Galerkin model*. Applied Acoustics 2020
4. H. Wang et al. *An arbitrary high-order discontinuous Galerkin method with local time-stepping for linear acoustic wave propagation*. JASA 2021

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